

2019 Southern Nevada Girls Math Tournament

Grade 8 Contest General Round

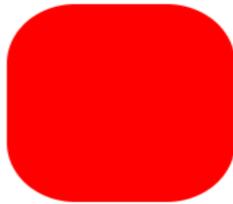
Name: _____

RULES

1. The General Round consists of 25 problems.
2. The General Round must be completed individually.
3. You will have 40 minutes to complete the General Round.
4. You will receive 1 point for each correct answer.
5. There is no penalty for incorrect answers.
6. You may NOT use a calculator on this round.
7. Answers are to be written on the provided lines. Units are not required.

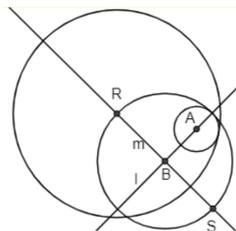
Score: ____/25 pts

1. _____ A man has a plan to ban cans from the grand Japan dam. I am not a fan of the man's plan to ban cans, so I cram and slam cans onto the dam. I also demand Denny's Grand Slams™ with Fanta cans. The man bans cans, so I slam and cram Fanta cans onto the dam. If the dam can handle 300 cans, and I am able to snag 10 cans per Grand Slam, and each Grand Slam costs \$10, how much money do I plan to spend to slam and cram cans onto the dam until it can't handle them?
2. _____ Write $\frac{1+3+9+27}{81+243+729+2187}$ as a common fraction.
3. _____ Call a currency "extraterrestrial" if it has coins in denominations $1, b, b^2$, and b^3 , measured in aliens. If the sum of all possible distinct 2 coin arrangements ($b + b$ would not be an arrangement) for an "extraterrestrial" currency is 255, find the value of the most valuable coin in aliens.
4. _____ Richard and Mortimer make their own gourmet anthropomorphic pickles. The number of pickles stuffed into a jar is inversely proportional to the shelf life of the pickles. This can be expressed as $L = \frac{K}{P}$ where L is shelf life in years, P is pickles in one jar, and K is some constant. If the Life of X pickles is $X+2$ years, and the Life of $X+6$ pickles is $X-2$ years (X is an integer), then what is the shelf life of $48x^2$ pickles? Express your answer as a common fraction.
5. _____ How many arrangements of SUBASUBA are there?
6. _____ How many non-congruent right triangles with positive integer side lengths have hypotenuse 25?
7. _____ A squirele is a square with four 90 degree circular arcs of a circle (with radius equal to the length of the straight part) cut off. If the radius of the circle is 1, find the perimeter of the figure.



8. _____ If $x + \frac{1}{x} = 3$, find $\gcd(x^3 + \frac{1}{x^3}, x^6 + \frac{1}{x^6})$
9. _____ Sid slowly slips across steps as she senselessly sips on soda. For each sip, Sid slips six steps south. Sid sips six sips per second, so she stops every sixty slips for six sips so she slips slower. If she should slip six hundred and six steps, how many sips did she sip?
10. _____ Everybody has a flashlight. Dig down thirty meters from the center of your house and you will find an unmarked wooden box. Inside will be a flashlight that can be modeled as a cylinder with radius 5 cm and height 10 cm, a frustum with bases of radius 5 cm and 7 cm and a height of 2 cm, and another cylinder with radius 7 cm and height 2 cm. It is up to you to decide how you will use the flashlight, but only under the condition that you can calculate the volume of the flashlight. What is the volume of the flashlight?

11. _____ Find the ratio of the radius of the incircle of a 3,4,5 right triangle and the circumradius of the same triangle.
12. _____ Miss Rhodesia is running a single elimination tournament where there are N teams. It is noticed that thirteen teams play at least seven games. Find the largest possible value of N .
13. _____ A magician tricks his audience into believing he can guess what card they pick by inconspicuously swapping out all of the cards in a normal deck for Kings of Hearts. However, this time he is getting cocky, so he picks a card without looking from the normal deck to keep in the trick deck, with the rest of the cards being replaced by Kings of Hearts. What is the probability the audience member does not pick a King of Hearts out of the trick deck on their first pick, ousting the magician as a fraud and forever tarnishing his reputation?
14. _____ Over all pairs (a,b) of three digit positive integers containing six distinct digits satisfying $a+b=1000$, find the maximum possible value of b .
15. _____ When Young LaFlama gains enough chakra, she enters Sicko Mode. When she enters Sicko Mode, LaFlama goes to the ranch, and creates a crop triangle. LaFlama creates an equilateral triangle ABC with side length 2, and appoints M the midpoint of BC . LaFlama, chooses points X and Y and traces them on AB and AC respectively such that triangle XMY is an isosceles right triangle with a right angle at M . What is the length of XY ?
16. _____ How many positive integers less than 10000 have digit sum 10?
17. _____ Find the perimeter of a the quadrilateral that, in order, has side lengths 3,5,8, and x if it has an incircle.
18. _____ A small circle A is internally tangent to a large circle B which has three times the radius of circle A . A line l connecting the centers of the circles is drawn. Then a line m perpendicular to l through the center of the larger circle is drawn, hitting the larger circle at points R and S . A circle R is such that the small circle is internally tangent to R . Find the radius of circle R if the radius of circle A is 1.



19. _____ Two eight-sided dice are on a table, one dice containing six ones and two random unknown, not necessarily distinct, numbers once and the other containing each of one

through eight once. John picks one of the two dice at random and rolls it twice, coming up on one both times. What's the probability John picked the biased dice?

20. _____ How many ways can one arrange four As, Bs, Cs in a line such that no As appear in the first four letters, no Bs appear in the next four letters, and no Cs appear in the last four letters?
21. _____ Suppose that real numbers satisfy $(xy+yz+zx)/(x+y+z) = xyz^{1/3}$ and $x^2+y^2+z^2 = 54$. If $x = y$, find x .
22. _____ Neo is fighting Agent Smith in the Matrix. The only way for Neo to beat Agent Smith is to become the one! To become the one, Neo must receive one as an output from this piecewise function. How many values could Neo input from 1-20 so that the function eventually ends up being a one? Example: Neo chooses $n = 4$. Neo receives 2, then 2 plugged in again receives 1.
{When n is even $f(n) = n/2$
{When n is odd $f(n) = n^2 - 1$
23. _____ Samuel picks five positive integers, none of which are the same, and creates ten new numbers by taking the positive difference of any two of the five numbers. Finally he generates a unique number by multiplying the ten numbers together. Find the largest positive integer that divides the unique number no matter the positive integers chosen.
24. _____ Samus Goodman is a master of number theory. Her student Matayo sees this and decides to give her combinatorics. Matayo says he has two special six sided die that have the property that the probabilities of getting any one of 2-12 when rolling both die is the same as it would be with normal die. Given that these special die are different than the normal 1-6 die, Matayo wants you to find out the sum of the squares of all the numbers on both die.
25. _____ Spencer the Ghost is a curious Mathcounts child who likes summing perfect squares. Amazingly, Spencer the Ghost correctly computes the number of such sums less than 1050 (where $a^2 + b^2$ is different than $b^2 + a^2$). If N is the number of triples, compute the three digit number that would be formed by the first three digits of N .

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Grade 8 Contest Target Round

Name: _____

RULES

1. The Target Round consists of 8 problems.
2. The Target Round must be completed individually.
3. You will have 6 minutes to complete each pair of questions.
4. The Target Round consists of 4 pairs, or 8 questions.
5. You will receive 2 points for each correct answer.
6. There is no penalty for incorrect answers.
7. You may use a calculator on this round.
8. Answers are to be written on the provided lines. Units are not required.

Score: ____/16 pts

1. _____ Find the largest positive integer that divides both $2^{15} - 1$ and $2^{20} - 1$.
2. _____ Find the product of the real values satisfying $(x^2 + 1)^3 + (x - 7)^3 = (x^2 + x - 6)^3$.

3. _____ Reignover, Huni, Red Velvet, JYP, Spencer Rhodes, Bang, Dahyun, Tzuyu, and the Higher Brothers are having a debate, about the answer to the problem, “TWICE creates a six-digit positive integer by writing the digit 7 in the hundred-thousands place, and then tossing a fair coin five times. If the coin comes up heads, he writes a 7 for the next digit; if the coin comes up tails, he writes a 0 for the next digit. What is the probability that TWICE’s number is divisible by 77? Express your answer as a common fraction.” Reignover says $\frac{1}{15}$, Huni says $\frac{5}{8}$, Red Velvet states $\frac{7}{16}$, JYP says 4, Spencer Rhodes says $\frac{6}{11}$, Dahyun says $\frac{5}{16}$, Bang says $\frac{7}{9}$, Tzuyu says $\frac{1}{5}$, and the Higher Brothers believe it is $\frac{17}{83}$. Who is correct?
4. _____ If $x + \frac{1}{x} = 3$, what is the value of $x^4 + \frac{1}{x^4}$?

5. _____ Huhueyhewtohuen has a perfectly spherical afro. If the radius of his afro grows at a rate of 1 inch per day, how large is his afro after 15 days if he started bald?
6. _____ I enjoy the act of eating dice. My friend also finds pleasure in consumption of dice. However, I only eat dice when I roll an odd number, and my friend only eats dice when she rolls a prime number. If we have 5 dice and we take turns rolling each die until one of us eats each die, eating the dice that meet our parameters, what is the probability that my friend eats 3 or more dice?

7. _____ There is a very tall man outside my window, standing on the ground. I live on the 6th floor of my apartment complex. His head is just tall enough so that it reaches the same height as my ceiling. The height of each floor in my complex, from floor to ceiling, is determined by the function where the n th floor has height of $\lfloor \frac{n^2}{3} \rfloor$ feet and there are 4 inches of insulation between floors. If the floor of the first floor is at the same height as ground level, how tall is the man outside my window in feet
8. _____ What is $\binom{0}{0} + \binom{1}{0} + \binom{1}{1} + \binom{2}{0} + \binom{2}{1} + \binom{2}{2} + \dots + \binom{20}{18} + \binom{20}{19} + \binom{20}{20}$?

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Grade 8 Contest Team Round

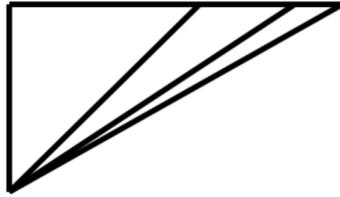
Team Name: _____

RULES

1. The Team Round consists of 10 problems.
2. The Team Round must be completed in teams of up to 4.
3. Your team will have 20 minutes to complete the Team Round.
4. Your team will receive 1 point for each correct answer.
5. There is no penalty for incorrect answers.
6. You may use a calculator on this round.
7. Answers are to be written on the provided lines. Units are not required.

Score: ____/10 pts

- _____ Find the smallest positive integer with 64 factors.
- _____ A right isosceles triangle with leg length 2 has infinite additional triangles with base length equal to half of the previous triangle constructed on the hypotenuse. What is the area of the figure?



- _____ On my way home, I saw the very tall man again. His legs are so long that his steps are 7 times longer than mine. However, after noticing the tall man behind me, I took steps at 3 times the rate of the tall man. If the tall man was 1.5 times farther away than my apartment door when I saw him, and the tall man was 5 feet away from me when I reached my apartment door, how many feet behind me was the tall man when I first noticed him?
- _____ A square on the coordinate grid has points: $(0,0)$, $(0,7)$, $(7,7)$, $(7,0)$. If there is a circle with a center at $(4,5)$ and a radius of 3, what are the points that the circle and the square intercept? hint: simplest radical form
- _____ I have found that the only way I can defeat my haters is by dabbing on them. However, I am unsure how many haters I have or how long it will take me to defeat them. My accountant and my secretary both are tasked with calculating how long it will take me to defeat my haters. They both know how many haters I have, but I do not know yet. If my accountant calculates how long it will take if I defeat 10 haters a day and my secretary calculates how long it will take if I defeat 100 haters a week, and my secretary calculates a number of days 60 days shorter than my accountant, how many haters do I have?
- _____ I'm attempting the "Write Math Problems at 3:00 A.M. Challenge". For each problem, I roll a pair of dice, if their product is prime, then I write an easy problem, if it is composite, then I write a difficult problem, and if it is neither, then I write an impossible problem. If I write 5 problems before I fall asleep, what is the probability that 4 or more of the problems are the same type? (Express your answer as a percent rounded to the nearest whole percent)
- _____ Spongebob's pants can be approximated by a rectangular prism with one of its faces with the largest area removed. Patrick's pants can be approximated by a hemisphere with the flat, circular face removed. Spongebob and Patrick are painting the outsides of their pants. If Spongebob's pants have side lengths 8 inches, 6 inches, and 4 inches, and Patrick's pants have radius $\frac{1}{2}$ feet, then what is the absolute difference in the amount of paint used by Spongebob and Patrick if they both only use one coat?
- _____ I have a very large amount of perfectly spherical marbles. The number of marbles I have is equal to the smallest number that leaves a remainder of 1 when it is divided by the first 20 positive integers greater than 1. If each marble has a radius of $\frac{1}{2}$ inches, what is the total volume of all my marbles in cubic inches?

9. _____ I have three kinds of dice, 4-sided, 6-sided, and 8-sided. Each n -sided die is marked with the first n positive integers. If I roll all three dice, what is the probability that their product equals a perfect power? (Where a perfect power is an positive integer raised to a power greater than 1) (Express your answer as a common fraction)
10. _____ I was standing on a coordinate grid at the origin. I walked an integral number of units chosen at random between 1 and 10, inclusive, to the right and an integral number of units chosen at random between 1 and 20, inclusive, upwards. If I am not an integral number of units away from the origin, what is the expected value of the area of the triangle formed by the path I walked and a straight line from my position to the origin to the nearest thousandth?